UFR Sciences Amiens, License de Physique S5

Outils Mathématique pour le Physique : TD-3

(I.Luk'yanchuk)

Exercice 1.

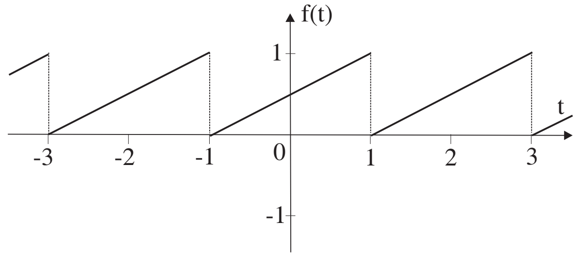
Demontrer que

$$\lim_{\lambda \to 0} \quad \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{x^2}{2\lambda^2}} \;=\; \delta(x)$$

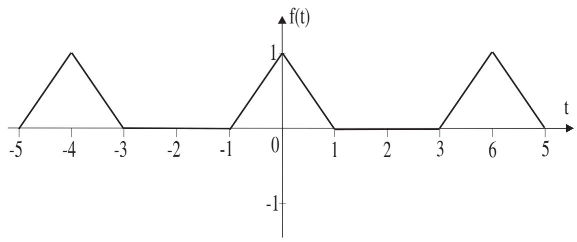
Exercice 2.

Exprimer les fonction suivante en séries de Fourrier

a)



b)*



SOLUTION

a) $\lim\limits_{\lambda\to 0}\frac{1}{\lambda\sqrt{2\pi}}e^{-\frac{x^2}{2\lambda^2}} \overset{x=0}{=} \lim\limits_{\lambda\to 0}\frac{1}{\lambda\sqrt{2\pi}} \overset{x=0}{=} \infty;$ $\lim\limits_{\lambda\to 0}\frac{1}{\lambda\sqrt{2\pi}}e^{-\frac{x^2}{2\lambda^2}} \overset{x\neq 0}{=} \lim\limits_{\lambda\to 0}\frac{1}{\lambda\sqrt{2\pi}}e^{-\frac{x^2}{2\lambda^2}} \overset{x=0}{=} 0;$ $\int_{-\infty}^{\infty}\frac{1}{\lambda\sqrt{2\pi}}e^{-\frac{x^2}{2\lambda^2}}dx = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-\frac{x^2}{2\lambda^2}}d\frac{x}{\sqrt{2}\lambda} = 1$

b) $f(t) = \frac{1}{2} + \frac{1}{2}t$, $T = 2$, $a_0 = \frac{2}{2}\int_{-1}^{1}\left(\frac{1}{2} + \frac{1}{2}t\right)dt = 1$, $a_n = 0$

$b_n = \frac{2}{2}\int_{-1}^{1}\left(\frac{1}{2} + \frac{1}{2}t\right)\sin\frac{2\pi n}{2}tdt = \frac{1}{2}\int_{-1}^{1}t\sin\frac{2\pi n}{2}tdt = \frac{1}{2\pi^2n^2}\left[\sin\pi nt - \pi nt\cos\pi nt\right]_{-1}^{1} = -\frac{\cos\pi nt}{\pi n} = \frac{(-1)^n}{\pi n}$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{(-1)^n}{\pi n}\sin\pi nt$$

b)* $f(t) = 1 - |t|$, $T = 4$, $b_n = 0$, $a_0 = \frac{2}{4}\int_{-1}^{1}f(t)dt = \frac{1}{2}$,

$a_n = \frac{2}{4}\int_{-1}^{1}(1 - |t|)\cos\frac{2\pi nt}{4}dt = -\int_0^1 t\cos\frac{\pi nt}{2}dt = -\frac{4}{\pi^2n^2}\left[\cos\frac{\pi nt}{2}\right]_0^1 - \frac{2}{\pi n}\left[t\sin\frac{\pi nt}{2}\right]_0^1 = \frac{4}{\pi^2n^2} - \frac{4}{\pi^2n^2}\cos\frac{\pi n}{2} - \frac{2}{\pi n}\sin\frac{\pi n}{2}$

si $n = 1, 2, 3, 4, ...$ $\cos\frac{\pi n}{2} = 0, -1, 0, 1, ...$ et $\sin\frac{\pi n}{2} = 1, 0, -1, ...$

$$\begin{aligned} f(t) &= \frac{1}{4} + \sum_{n=1}^{\infty}\left(\frac{4}{\pi^2n^2} - \frac{4}{\pi^2n^2}\cos\frac{\pi n}{2} - \frac{2}{\pi n}\sin\frac{\pi n}{2}\right)\cos\frac{\pi n}{2}t \\ &= \frac{1}{4} + \sum_{n=1}^{\infty}\frac{4}{\pi^2n^2}\cos\frac{\pi n}{2}t - \sum_{k=1}^{\infty}\frac{(-1)^k}{\pi^2k^2}\cos\pi kt - \sum_{k=0}^{\infty}\frac{(-1)^k}{\pi(2k+1)}\cos\frac{\pi(2k+1)}{2}t \end{aligned}$$